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The concentration fields of disperse impurity are calculated on the basis of a $k-\epsilon$ model of turbulence in a wide range of Stokes numbers.

Calculating the concentration fields of disperse impurity is one of the most important problems in the numerical modeling of two-phase turbulent jets. The character of the impurity distribution in the jet for the case of solid particles is determined by two basic mechanisms: turbulent diffusion of particles and their migrational transfer, which is due to the initial rotation of the particles, acquired in impacts on the tube wall, and the presence of phase slipping. One characteristic of the first mechanism is the Schmidt number Sc = v_t/v_t D_{D} , for which a theoretical dependence was obtained for the first time in [1] within the framework of the theory of Prandtl mixing paths. In [2], it was shown that, in taking account of the initial particle velocity of the particles incident in a turbulent mole, an analog of the Schmidt number may be obtained for a two-phase flow, permitting transition in the limit to the Schmidt number for a gas with reduction in particle size. However, calculation by this model gives an underestimate of the damping rate of the axial concentration distribution of the impurity, which has prompted the construction of various approaches determining the diffusional properties of a heavy impurity in a turbulent jet [3]. As shown by experiment [4], migrational transfer of impurity, causing the phenomena of concentration and dispersion, dominates in the initial section of the jet, and may be described by taking account of Magnus forces and radial phase slipping [5]. Nevertheless, the calculation scheme for a two-phase turbulent jet proposed in [5] is sufficiently cumbersome: calculation of the two-phase jet is preceded by solution of the problem of the emission of a one-phase jet, and then the turbulent transfer coefficients are found, taking account of the influence of particles on the theory [2]. Models of second-order turbulence are promising for the calculation of two-phase jets.

In the present work, the distribution of concentration fields of the disperse phase in a turbulent jet on the basis of a two-parameter model is investigated numerically, using the transfer equations of pulsational energy and the rate of its dissipation. The dynamic phase interaction is determined, as in [5], by the drag force and the Magnus force. The system of equations for the mean quantities describing the emission of an axisymmetric turbulent gasdisperse jet, taking account of phase slipping and particle rotation, takes the form

$$\frac{\partial u_g}{\partial x} + \frac{1}{y} \frac{\partial}{\partial y} (y v_g) = 0, \tag{1}$$

$$\frac{\partial}{\partial x} \left(\rho_p u_p \right) + \frac{1}{y} \frac{\partial}{\partial y} \left(y \rho_p v_p^* \right) = 0, \tag{2}$$

$$\rho_g \left(u_g \frac{\partial u_g}{\partial x} + v_g \frac{\partial u_g}{\partial y} \right) + \frac{1}{y} \frac{\partial}{\partial y} \left(y \rho_g \langle u_g \dot{v}_g \rangle \right) = -F_x, \tag{3}$$

$$\rho_p\left(u_p \frac{\partial u_p}{\partial x} + v_p^* \frac{\partial u_p}{\partial y}\right) + \frac{1}{y} \frac{\partial}{\partial y} \left(y \rho_p \langle u_p^{\prime} v_p^{\prime} \rangle\right) = F_x, \tag{4}$$

$$\rho_p\left(u_p \frac{\partial v_p}{\partial x} + v_p^* \frac{\partial v_p}{\partial y}\right) + \frac{1}{y} \frac{\partial}{\partial y} \left[y\left(\rho_p \langle v_p^{\prime 2} \rangle + v_p \langle \rho_p^{\prime} v_p^{\prime} \rangle\right)\right] = F_y, \tag{5}$$

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$$\rho_p\left(u_p\frac{\partial\omega_p}{\partial x}+v_p^*\frac{\partial\omega_p}{\partial y}\right)+\frac{1}{y}\frac{\partial}{\partial y}\left[y\left(\rho_p\langle v_p^*\omega_p^*\rangle+v_p\langle \rho_p^*\omega_p^*\rangle\right)\right]=f,$$
(6)

$$u_g \frac{\partial k}{\partial x} + v_g \frac{\partial k}{\partial y} = \frac{1}{y} \frac{\partial}{\partial y} \left(y \frac{v_t}{\sigma_k} \frac{\partial k}{\partial y} \right) + P_k - \varepsilon - \varepsilon_p, \tag{7}$$

$$u_g \frac{\partial \varepsilon}{\partial x} + v_g \frac{\partial \varepsilon}{\partial y} = \frac{1}{y} \frac{\partial}{\partial y} \left(y \frac{\mathbf{v}_t}{\sigma_{\varepsilon}} \frac{\partial \varepsilon}{\partial y} \right) + \frac{\varepsilon^2}{k} \left(c_{\varepsilon_1} \frac{P_k}{\varepsilon} - c_{\varepsilon_2} + c_{\varepsilon_3} \chi \right) - \Phi_p; \tag{8}$$

$$\varepsilon_{p} = \frac{1}{\rho_{g}} \sum_{i} \langle V_{gi}F_{i}^{'} \rangle, \quad \Phi_{p} = \frac{2v}{\rho_{g}} \sum_{ij} \left\langle \frac{\partial V_{gi}}{\partial x_{j}} \frac{\partial F_{i}^{'}}{\partial x_{j}} \right\rangle,$$
$$v_{p}^{*} = v_{p} + \frac{\langle \rho_{p}^{'}v_{p}^{'} \rangle}{\rho_{p}}, \quad P_{h} = v_{t} \left(\frac{\partial u_{g}}{\partial y}\right)^{2},$$
$$f = \frac{10}{3} \beta \rho_{p} \Omega, \quad \Omega = -\omega_{p} + \frac{1}{2} \frac{\partial u_{g}}{\partial y}.$$

If Eqs. (3), (5), and (6) are written in dimensionless form, the projection of the dimensionless drag force and the right-hand side of Eq. (6) take the following form:

$$\overline{F}_{\mu i} = \frac{1}{\text{Stk}} \left(1 + b_1 \operatorname{Re}_p^{1/2} + b_2 \operatorname{Re}_p \right) \overline{\rho}_p \overline{V}_{ri}, \tag{9}$$

$$\overline{f} = \frac{3.33}{\text{Stk}} \,\overline{\rho}_p \,\overline{\Omega}. \tag{10}$$

It is evident from Eqs. (9) and (10) that, with increase in Stokes number, i.e., with increase in inertia of the impurity, the terms \bar{f} , $\bar{F}_{\mu i}$ characterizing the phase interaction in Eqs. (3)-(6) decrease.

The usual relations of the $k\!-\!\varepsilon$ model are used to represent the turbulent frictional stress

$$\langle u'_{g} v'_{g} \rangle = -v_{t} \frac{\partial u_{g}}{\partial y}, \quad v_{t} = c_{\mu} \frac{k^{2}}{\varepsilon},$$
 (11)

as modified in [6] for the case of an axisymmetric jet by taking account of the mechanism of eddy extension using the invariant

$$\chi = \frac{1}{4} \left(\frac{k}{\varepsilon}\right)^3 \left(\frac{\partial u_g}{\partial y}\right)^2 \frac{v_g}{y}.$$
 (12)

Suppose that the constants of the single-phase model of turbulence [6] c_{μ} , σ_k , σ_{ϵ} , c_{ϵ_1} , c_{ϵ_2} , c_{ϵ_3} retain their values unchanged.

The gradient representation of the correlational moment characterizing the turbulent mass transfer of the disperse phase in the transverse direction

$$\langle \rho_{p} v_{p} \rangle = -D_{p} \frac{\partial \rho_{p}}{\partial y},$$
 (13)

requires the determination of the transverse turbulent diffusion coefficient. The expression for D_p obtained by the method of [7-9], taking account of phase slipping and particle rotation, takes the form

$$D_{p} = \langle v_{g}^{\prime 2} \rangle \left[\frac{1}{\varphi_{yy}} + \frac{1}{\varphi_{xx}} \left(\frac{\gamma_{yx}}{\gamma_{yy}} \right)^{2} \right] + \frac{2\gamma_{yx}}{\gamma_{yy}\varphi_{xy}} \langle u_{g}^{\prime} v_{g}^{\prime} \rangle, \qquad (14)$$

$$\gamma_{ii} = \gamma + \gamma_{0} \frac{V_{ri}^{2}}{V_{r}^{2}}, \quad \gamma_{ij} = \gamma_{0} \frac{V_{ri}V_{rj}}{V_{r}^{2}} + (-1)^{\delta_{iy}} \lambda\Omega,$$

$$\gamma = \beta \left(1 + b_{1} \operatorname{Re}_{p}^{1/2} + b_{2} \operatorname{Re}_{p}\right), \quad b_{1} = 0.179,$$

$$\gamma_{0} = \beta \left(0.5 b_{1} \operatorname{Re}_{p}^{1/2} + b_{2} \operatorname{Re}_{p}\right), \quad b_{2} = 0.013.$$

Note that for very small particles $\gamma_{yx}/\gamma_{yy} \rightarrow 0$, $\varphi_{yy} \rightarrow 1/T_L$, so that the expression for the turbulent diffusion coefficient of the particles transforms to the well-known expression for the transverse diffusion coefficient of the liquid element $D_g = \langle v'^2_g \rangle T_L$ [7].

The correlation of the pulsation velocities of the disperse phase and also the additional dissipative terms ε_p , Φ_p appearing in the transfer equation for the pulsational quantities are found analogously to [8-11], taking account of the phase slipping and particle rotation.

The boundary conditions at the jet axis and in the submerged space take the form

$$y = 0: \quad \frac{\partial u_g}{\partial y} = \frac{\partial u_p}{\partial y} = \frac{\partial \rho_p}{\partial y} = \frac{\partial k}{\partial y} = \frac{\partial \varepsilon}{\partial y} = v_p = v_g = \omega_p = 0, \quad (15)$$

$$y = \infty: \ u_g = u_p = \rho_p = k = \varepsilon = \frac{\partial v_p}{\partial y} = \frac{\partial \omega_p}{\partial y} = 0.$$
(16)

The boundary conditions in the initial cross section of the two-phase jet relative to u_g , k, ε are found from the solution of the auxiliary problem of carrying-medium motion in the stabilized section of the circular tube in the presence of particles. The dispersephase parameters u_p , ρ_p are specified from experiment; v_p , ω_p are selected in the calculation process, taking account of estimates [12] from the condition of best agreement with experimental data on the distribution of disperse impurity in the jet, analogously to [5].

The system in Eqs. (1)-(8), with the corresponding boundary conditions, is solved numerically by the finite-difference method, using an implicit six-point scheme of second-order accuracy [13]. Numerical investigations show [10] that the two-parameter model here proposed allows concentration and dispersion effects of a sufficiently large impurity to be described, analogously to [5].

The distribution along the jet axis of the maximum ratio ω_p/ω_{p0} in this flow is shown in Fig. 1 for various Stokes numbers. It is evident that, when Stk < 1, no account may be taken of the influence of particle rotation on their scattering; correspondingly, Eq. (6) may be omitted. At the same time, experimental data show [4] that, in the scattering of small impurity, the concentration effect is also observed. Using the model of [5], pronounced increase in the concentration of small particles at the jet axis cannot be obtained, in view of the rapid damping of ω_p , as follows from Fig. 1. In this case, as shown by calculation, concentration of the impurity is due to the strong inhomogeneity of the turbulent-energy field in the initial section of the jet.

In Eq. (5), there is a term which may be written in the following form at small Stokes numbers:

$$R = \frac{1}{y} \frac{\partial}{\partial y} \left(y \rho_p \langle v_p^2 \rangle \right) \approx \frac{1}{y} \frac{\partial}{\partial y} \left(y \rho_p \frac{\gamma_{yy}}{\varphi_{yy} + \gamma_{yy}} \frac{2}{3} k \right).$$

In [14], it was shown that R causes migrational transfer of input impurity in the direction of decrease in intensity of the pulsational-velocity field of the gas. In fact, if R is introduced on the right-hand side of Eq. (5), R may be regarded as an additional force proportional to $(-\partial k/\partial y)$. When Stk > 15, it may be assumed that R \approx 0, since $\gamma_{yy}/\varphi_{yy} \rightarrow 0$; impurity scattering close to the end of the tube in this case is determined by migration under the action of the Magnus force.

When Stk < 1, not only turbulent diffusion but also migration in the direction of decrease in turbulent energy plays a pronounced role close to the end of the tube. Note that, in calculations, this mechanism ensures concentration of the fine impurity, but to a much lesser extent than is observed experimentally.

At intermediate Stokes numbers 1 < Stk < 15, both the migration mechanisms must be taken into account. Comparison of the experimental data of [4] and the profiles of impurity flow rate calculated taking account of the two migration mechanisms at Stk = 4.6 is shown in Fig. 2.

In the remote region of the jet, where migrational transfer is small, the scattering of disperse impurity is determined primarily by the turbulent diffusion. In Fig. 3, experimental data on the scattering of disperse phase along the jet axis [3] are shown, together



Fig. 1



Fig. 1. Damping of the angular velocity of impurity rotation along the jet axis when $\kappa_0 = 0.5$: 1) Stk = 1; 2) 4; 3) 13; 4) 35; 5) 80.

Fig. 2. Profile of impurity flow rate $\kappa_0 = 0.47$, Stk = 4.6 in cross sections $x/r_0 = 0$ (1), 2.86 (2), 11.43 (3), 17.14 (4).

Fig. 3. Change in the distributed density of the disperse impurity along the jet axis: 1) $\kappa_0 = 0.46$, Stk = 0.8; 2) 0.56, 18. Calculation: curves with $c_2 = 1$; dashed curve with $c_2 = 0.5$.

TABLE 1. Average Schmidt Numbers over the Jet Cross Section at Stk = 105, $\kappa_{\rm g}$ = 0.3

x/r ₀	Expt. α [15]	Calc.		
		Sc ^o	Sc [°]	Scp
35 40 45 50 55	2,33 2,38 2,44 2,51 2,53	2,20 2,24 2,30 2,38 2,41	2,98 3,14 3,27 3,39 3,52	2,50 2,57 2,64 2,72 2,98

with results of numerical calculation. For small impurity Stk = 0.8, good agreement with experimental data is observed; larger impurity Stk = 18 is scattered much more rapidly in the experiment than in calculations.

In [7, 10], the turbulent-diffusion coefficient of the particles is obtained on the basis of approximation of the space-time correlation of the velocity of the medium along the particle trajectory using the dependence

$$R_{xx}(\tau) = \exp\left(-a\tau\right) R_E\left(\left|u_g - u_p\right|\tau\right),\tag{17}$$

$$D_p = \frac{2}{3} k \Lambda_E \frac{\sqrt{2/3k} + 0.5 |u_g - u_p|}{(\sqrt{2/3k} + |u_g - u_p|)^2}, \qquad \Lambda_E = c_1 k^{3/2} / \epsilon.$$
(18)

The value of the empirical constant $c_1 = 0.17$ is chosen by comparison of the results of calculation with experimental data on the scattering of low-inertia impurity Stk < 1 in the remote region of the jet, where the influence of migrational effects may be neglected.

Calculations using Eqs. (14) and (17) give underestimates in comparison with experimental values of the axial impurity concentration in the remote region of the jet for sufficiently inertial particles (in Fig. 3, Stk = 18). This indicates an overestimated influence of the mean phase slipping on the turbulent-diffusion coefficient of the particles. In fact, with increase in Stk, there is an increase in mean phase slipping; as is evident from Eq. (18), D_p decreases here. The decrease in D_p with increase in $|ug - u_p|$ is described qualitative-ly correctly by Eq. (18), but the rate of decrease in D_p should not be so rapid.

It may be assumed that the dependence of $R_{XX}(\tau)$ on $|u_g - u_p|$ is more complex than according to Eq. (17). In the first approximation, suppose that

$$R_{xx}(\tau) = \exp(-a\tau) R_E(c_2 | u_g - u_p | \tau).$$
(19)

Then the turbulent diffusion coefficient of the particles may be written in the form

$$D_{p} = \frac{2}{3} \frac{k^{5/2}}{\varepsilon} c_{1} \frac{\sqrt{2/3k} + 0.5c_{2}V_{r}}{(\sqrt{2/3k} + c_{2}V_{r})^{2}} + \frac{2\gamma_{yx}}{\gamma_{yy}\varphi_{xy}} \langle u_{g}^{*}v_{g}^{*} \rangle.$$
(20)

Experimental data on the scattering of inertial impurity [3, 5] with the new empirical constant $c_2 = 0.5$ are in satisfactory agreement with the calculation results. In Fig. 3, the dashed curve shows this calculation for Stk = 18. Note that, in the scattering of inertial impurity Stk > 10 at large distances from the end of the tube, the Magnus force exerts an influence; the measurements in the jets of [3-5], however, were made at distances no greater than $x/r_0 = 60$. In connection with this, the empirical constant c_2 must be refined on the basis of measurements in the remote region of two-phase jets.

In [1-3], the Schmidt number was taken as the basic characteristic in determining the turbulent diffusion coefficient of the particles. In the present case, it may be calculated using Eqs. (11) and (20): Sc = v_t/D_p . The mean Schmidt number over the jet cross section may be determined as follows:

$$Sc^{\circ} = \frac{2}{L^2} \int_0^L y \, Sc \, dy. \tag{21}$$

The values of Sc° calculated in this way for various Stokes numbers are in good agreement with the experimental ratio [15]: $\alpha = y_u/y_k$.

Table 1 gives values of α and mean values of the Schmidt number for various jet cross sections: Sc°, calculated using Eq. (19); Sc[°]_x, calculated using Eq. (17); Sc[°]_p, from the theory of [2]. The result of the calculation with the additional empirical constant c₂ is in good agreement with the experimental data; all the quantities behave qualitatively in an analogous manner.

It is of interest to compare the results of calculating the turbulent-diffusion coefficient according to the method proposed here and according to the theory of [1, 2]. In the present model and in [1, 2], the influence of particles on the pulsational characteristics is taken into account differently. To eliminate the influence of this factor, concentrating attention on the turbulent diffusion coefficient of the particles, the impurity concentration is assumed to be very small. Then the formula of [2] for the Schmidt number may be simplified:

$$Sc_{n}/Sc_{\sigma} = 2/(2 - n - n_{0}).$$
 (22)

The quantity characterizing the potential for particle entrainment as $\rho_{p} \neq 0$ is determined from the equation

$$\ln |n/n_0| = -2A/|2 - n - n_0|, \ A = \beta l_u / v'_{\sigma_0}, \tag{23}$$

where the dimensionless parameter A, in contrast to the Stokes number, is a local ratio of the turbulent time scale and the particle relaxation time β^{-1} . The initial value is found from the formula of [2] as $\rho_p \rightarrow 0$:

$$n_0 = \frac{1}{e-2} \left[\frac{1 - \exp(-A_k)}{A_k} + \frac{1 - \exp(1 - A_k)}{1 - A_k} \right], \ A_k = 0, 1 \ A.$$



Fig. 4. Variation in Schmidt number when $\overline{V}_r = 0$: by the method of [7] (1) and according to Eq. (22) (4); and when $\overline{V}_r = 0.3$: with $c_2 = 0.5$ (2), 1 (3), and according to the theory of [2, 16](5).

The results of calculating the ratio of the Schmidt numbers of the particles and the gas $Sc_p/Sc_g = D_g/D_p$ as a function of A, i.e., the relative inertia of the particles, are shown in Fig. 4. It is evident that all the curves give the same qualitative description of the change in Schmidt number of the disperse phase; with decrease in inertia of the particles (increase in A), $Sc_p \rightarrow Sc_g$, which is equivalent to $D_p \rightarrow D_g$. All the models predict a decrease in turbulent diffusion coefficient of the particles on taking account of the mean phase slipping. The introduction of the empirical constant c_2 in the present model allows the slowing in decrease in D_p with increase in mean phase slippage which was seen earlier in experimental data to be included in the theoretical results.

Thus, the two-parameter model of a gas-disperse turbulent jet proposed here allows the basic mechanisms of concentration-field formation of the disperse impurity to be described in the Stokes-number range 0.5 < Stk < 100.

NOTATION

x, y, coordinates; u, v, mean velocity components along the x and y axes; u', v', pulsational velocity components; F, phase-interaction force; δ , particle diameter; V_r, modulus of the difference in mean phase velocities; v, v_t , kinematic molecular and turbulent viscosity; k, ε , kinetic energy of turbulent pulsations and rate of its dissipation; r_0 , tube radius; β^{-1} , particle relaxation time; ρ^0 , ρ , true and distributed density; ε_p, ϕ_p , additional dissipative terms; Pk, generation of turbulent energy; D, turbulent diffusion coefficient; R_{XX} , space-time correlation of the gas velocities along the particle trajectories; R_E , Euler spatial correlation; TL, TE, AE, Lagrangian time scale, Euler time scale, and spatial scale of turbulence; l_u , mixing path with respect to gas velocity, a, ϕ_{ij} , exponential indices; t, τ , time; g = ρ_{pup}/ρ_{gug} , specific flow rate of particles; κ_0 , ratio of particle flow rate to gas flow rate at the end of the tube; χ , invariant; L, jet radius; yu, y_K, ordinates of jet at which half the axial gas velocity and particle concentration is reached; $n = (v'_g - v'_g)$ $v'_p)/v'_{g_0}$, v'_{g_0} , initial gas velocity in mole; A, dimensionless quantity characterizing the inertia of a particle; $Re_p = \delta V_r/v$, Reynolds number; $Sc = v_t/D_p$, Schmidt number; Stk = $\rho_p^{\delta^2} u_{gZ}/(36 \nu \rho_{gT_0})$, Stokes number; ω_p , angular velocity of particle rotation; Ω , relative angular velocity; c_{μ} , σ_k , σ_{ϵ} , c_{ϵ_1} , c_{ϵ_2} , c_{ϵ_3} , c_1 , c_2 , empirical constants; $\lambda = 0.75 \rho_0^0 \rho_0^0 \rho_0$, Yij, Y, Y₀, b, b₁, b₂, coefficients. Dimensionless quantities: $\bar{V}_r = V_r u_{gz}$; $\bar{\Omega} = 2r_0 \bar{\Omega}/u_{gz}$; $\bar{\rho}_p = \rho_p / \rho^0 g$; $f = 4r_0^2 f / (\rho^0 g u^2 g z)$; $\bar{F}_{\mu i} = 2r_0 F_{\mu i} / (\rho^0 g u^2 g z)$; f, moment of phase interaction; u_{gz} , gas velocity at the tube axis at end of tube. Indices: g, p, parameters of gas and disperse phase; m, at the jet axis; 0, at the end of the tube; z, at the tube axis at the end of the tube.

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EFFICIENCY OF ACCELERATING TUBES OF JET GRINDING MILLS

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An analysis is made of the energy efficiency of accelerating tubes in counterflow jet grinding mills. The dependence of the efficiency of these tubes on the parameters of the two-phase flow is established.

The grinding of solid materials is one of the most energy-intensive processes in industry. This fact makes it particularly important to select the proper method of grinding for a given case. Thus, analysis of the energy efficiency of grinding mills is of definite interest with regard to improving grinding technology and mill design. One promising trend in grinding is the use of jet mills [1, 2], in which the material is ground by high-speed impact. Gas or steam is usually used as the working substance, the energy of the gas or steam accelerating the starting material to velocities at which it breaks up upon impact against an obstacle (the wall of the mill or another portion of the material being ground). Here, the energy of the working substance is spent on the completion of useful work in accelerating particles of the material being ground, as well as on irreversible losses connected with the evolution of heat in interphase friction. Both types of energy expenditures depend on the phase slip

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